View Factors for an Infinite Rectangular Duct Enclosing a Transparent Medium by the Discrete-Ordinates Method

K. H. Byun*, Theodore F. Smith and A. Sanchez*****

(Received July 2, 1996)

The purpose of this study is to compute the view factors for an infinite rectangular system by the discrete-ordinates method. The effects of number of spatial divisions, number of discrete-ordinate directions, and the values of the weighting factors on the accuracy of the solution are studied. If shading appears, the accuracy of the solution depends both on the number of discrete ordinate directions and number of spatial divisions. Three different weighting factor sets, equal weighting factors for each direction cosine, Chevyshef quadratures and weighting factors from the trapezoidal integration rule are tested. The results are not sensitive to the choice of three different weighting factor sets.

Key Words: View Factors, Infinite Rectangular Duct, Discrete-Ordinates Method, Finite Difference Weighting Factor

Nomenclature

F_{S-N} : View factor from the south wall to the north wall
H, H_1 : Height of the system and protrusion
I : Intensity
L, L_1 : Width of the system and protrusion
M : Number of division for one quadrant
n : Number of spatial division
q : Radiative heat flux, W/m^2
T : Temperature, K

Greeks

α : Finite difference weighting factor (Sanchez and Smith, 1991)
η_i, μ_i : y and, x-direction cosine
σ : Stefan-Boltzman constant
ω_i : i-discrete ordinate weighting factor

Subscripts

b : Blackbody
N, S, E, W : North, south, east, west
i : i-discrete ordinate direction

* Department of Mechanical Engineering, Dongguk University, Seoul, Republic of Korea
** Department of Mechanical Engineering, The University of Iowa, Iowa City, U. S. A.
*** Escuela de Ingenieria Mecanica, Universidad de los Andes, Merida, Venezuela

1. Introduction

There are many applications of surface-to-surface radiative heat exchange in the transparent media such as heating and cooling of raw or finished materials, drying, heat transfer in rooms and cooling of the electronic components. Many kinds of methods appear in the literature to analyze the radiative heat exchange in these systems. For example, the methods used are such as the method utilizing the view factor (Hottel and Sarofim, 1967), Monte Carlo method (Siegel and Howell, 1981) and the Stochastic method (Naraghi and Chung, 1984).

Results on the view factors for various geometry have continuously been reported by many researchers. View factors can be calculated by direct integration, Monte Carlo method (Siegel and Howell, 1981), the ray tracing method (Baumeister, 1990), and the crossed-string method (Hottel and Sarofim, 1967 ; Siegel and Howell, 1981). Sanchez and Smith (1991) proposed that the view factors can be obtained by the discrete-ordinates method. The discrete-ordinates method is usually used for the analysis of the radiative heat transfer in participating media (Chandrasekhar, 1950 ; Fiveland, 1984, 1988 ; Kim and Lee, 1988 ; Kim and Baek, 1991 ; Truelove, 1987).

Sanchez and Smith (1991) published the radiative heat transfer results for a two-dimensional rectangular enclosure filled with a transparent medium. The system may have protrusions or inserted rectangular material. The effects of the finite-difference weighting factor, grid pattern, number of discrete ordinates angle, effect of surface emittance are studied. However, the results on the view factors are not published yet.

The primary purpose of this study is to find the view factors of an infinite rectangular duct system by using the discrete-ordinates method. The secondary purpose is to obtain the value of the finite difference weighting factor that certifies the exact solution with increase of the number of spatial and angular divisions. It is known that the finite -difference weighting factor (Fiveland, 1984, 1988 ; Kim and Lee, 1988 ; Kim and Baek, 1991 ; Truelove, 1987) is meaningful in analyzing radiation through participating media. The question is whether the finite difference weighting factor can be utilized for analyzing a system enclosing transparent media. The third purpose is to study the effect of the different sets of discrete-ordinates weighting factors on the accuracy of the results.

2. Analysis

2.1 Governing equations

The present configuration is a two-dimensional rectangular duct as depicted in Fig. 1 with the coordinate system. All the bounary surfaces are assumed opaque and black. The medium in the duct is nonparticipating. It is assumed that the heat transfer occurs only by thermal radiation. The length and height of the system are L and H, respectively. The protrusion length and heights are L_1, H_1, respectively.

The arbitrary control volume that appears in Fig. 2 is generated by dividing the length by n_j and the height by n_i. The center point of the control volume is designated as P. Subscripts N, S, E, W appear on the north, south, east and west

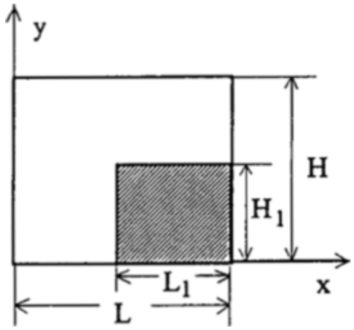

Fig. 1 System description

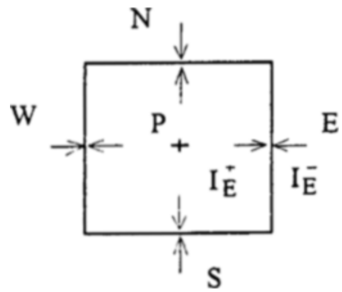

Fig. 2 Control volume description

surfaces with respect to the point P, respectively. A positive superscript (+) is assigned to the intensity coming out from the control volume and a negative superscript (−) is assigned to the intensity entering the control volume. The rotation angle 2π is equally divided at the point P. In this paper, each of 4 quadrants is divided by M angular directions, thus, there are 4 M discrete ordinate directions.

The radiative transport equations for an arbitrary i-discrete direction is

$$\frac{dI_i}{ds}=\mu_i\frac{\partial dI_i}{\partial x}+\eta_i\frac{\partial dI_i}{\partial y}=0 \qquad (1)$$

I is the intensity and s is the length of travel. μ_i and η_i is x- and y-direction cosines.

For a transparent medium, the divergence of radiative flux for i-discrete direction is expressed as

$$\frac{\partial q_{ix}}{\partial x}+\frac{\partial q_{iy}}{\partial y}=0 \qquad (2)$$

where q_{ix} and q_{iy} are the radiative flux in the x - and y-direction, respectively. The radiative fluxes expressed in terms of intensity are:

$$q_{ix}=\mu_i I_i,\ q_{iy}=\eta_i I_i \tag{3}$$

If Eq. (1) is discretized by using the intensity at the point P and the intensities at the west and south boundaries, then Eq. (1) is transformed to

$$\mu_i(I_{iP}^{+}-I_{iW}^{-})\Delta y+\eta_i(I_{iP}^{+}-I_{iS}^{-})\Delta x=0 \tag{4}$$

where Δx and Δy are the control volume length and height, respectively. In case $\Delta x=\Delta y$, Eq. (4) reduces to

$$I_{iP}^{+}=\frac{\mu_i I_{iW}^{-}+\eta_i I_{iS}^{-}}{\mu_i+\eta_i}=\frac{\mu_i I_{iE}^{-}+\eta_i I_{iN}^{-}}{\mu_i+\eta_i} \tag{5}$$

The last term in Eq. (5) is obtained by discretizing Eq. (1) by using the intensity at the point P and the intensities at the east and north boundaries. If Eq. (2) is discretized by utilizing Eq. (3) and the P, W, S, E, N intensities, the resulting equation is the same as Eq. (5). Thus, Eq. (5) satisfies both Eqs. (1) and (2).

If the radiative heat flux at east surface is expanded about the radiative heat flux at the point P by Taylor series, then

$$q_{iE}=q_{iP}+\frac{\partial q_{iP}}{\partial x}\frac{\Delta x}{2} \tag{6}$$

After spatially discretizing the differential terms in Eq. (6) by using q_E and q_W, Eq. (3) is substituted into Eq. (6). It follows that

$$I_{iE}^{+}=I_{iP}^{+}+\frac{I_{iE}^{-}-I_{iW}^{-}}{2}=2I_{iP}^{+}-I_{iW}^{-} \tag{7a}$$

The following equations are obtained by using a similar procedure as described in Eq. (7a).

$$I_{iN}^{+}=2I_{iP}^{+}-I_{iS}^{-} \tag{7b}$$

$$I_{iW}^{+}=2I_{iP}^{+}-I_{iE}^{-} \tag{7c}$$

$$I_{iS}^{+}=2I_{iP}^{+}-I_{iN}^{-} \tag{7d}$$

The positive intensities at the point P become the negative intensities at the adjacent control volumes since these are incoming fluxes. The Eq. (7) is the same as the intensity relations in the paper by Sanchez and Smith (1991) if the value of the finite difference weighting factor α is 0.5. Thus, as long as $\Delta x=\Delta y$ and the point P is located at the center of the control volume, the value of the finite difference weighting factor α should be 0.5.

2.2 Boundary conditions

If the boundary walls are opaque and black, the radiative intensity of the i-discrete direction is

$$I_{i,wall}^{+}=\frac{\sigma T_{wall}^4}{\pi}\qquad \text{wall}=\text{N, S, E, W} \tag{8}$$

where σ is the Stefan-Boltzmann constant and T is the absolute temperature.

2.3 The radiative heat flux

The radiative flux q_{wall} at each boundary surface are as follows:

$$q_{wall}=\sum_{i=1}^{2M}\mu_i\omega_i(I_i^{+}-I_i^{-})\qquad \text{wall}=\text{E, W} \tag{9a}$$

$$q_{wall}=\sum_{i=1}^{2M}\eta_i\omega_i(I_i^{+}-I_i^{-})\qquad \text{wall}=\text{N, S} \tag{9b}$$

where ω_i is the weighting factor for i-discrete ordinate direction.

2.4 Discrete ordinate directions and weighting factors

2.4.1 The method by Sanchez and Smith (1991)

The discrete ordinate direction angles are generated by dividing the quadrant polar angle $\pi/2$ by M (Sanchez and Smith ; 1991).

$$\phi_i=\frac{\Delta\phi}{2}+(i-1)\Delta\phi,\quad \Delta\phi=\frac{\pi}{2M}$$
$$i=1,\ M \tag{10}$$

By assuming the equal weighting factor for each direction, the values of weighting factors are (Sanchez and Smith ; 1991):

$$\omega=\frac{\pi}{2\sum_{i=1}^{M}\cos\phi_i} \tag{11}$$

2.4.2 Chevyshef quadrature

This set is also based on equal weights and equal angular increments. Discrete angles are the same as in Eq. (10). The weighting factor of Chevyshef quadrature (Atkinson; 1978) is obtained by dividing the quadrant polar angle $\pi/2$ by M.

$$\omega=\frac{\pi}{2M} \tag{12}$$

2.4.3 Trapezoidal method

Discrete angles have an equal increment defined by

$$\phi_i=(i-1)\Delta\phi, \quad \Delta\phi=\frac{\pi}{2M}$$
$$i=1, M+1 \tag{13}$$

There are M+1 discrete-ordinate directions including two of quadrant boundary directions. The values of the weighting factors used for this case are as follows (Atkinson ; 1978). Equal weights are assigned for the inner direction weighting factors. The weighting factors for the boundary directions are a half of the inner weighting factors.

$$\omega_i=\frac{\pi}{2M}, \quad i=2, M$$
$$\omega_i=\frac{\pi}{4M}, \quad i=1 \text{ or } M+1 \tag{14}$$

2.5 Numerical solution procedure

It is assumed that entering intensities at the system boundary walls are given.

(1) Starting from the control volume at the corner of the system boundary, choose one arbitrary discrete ordinate direction.

(2) Compute the intensities leaving the control volume by using Eqs. (5) and (7). The intensities leaving the control volume become the intensities entering two neighboring control volumes. Continue this procedure until all the control volumes are swept.

(3) Repeat the steps from (1) to (2), starting from remaining three corner control volumes.

(4) At each control volume, find the value of the radiative flux by Eq. (3). This step may be done at the step (6) if there is no storage problem.

(5) Repeat the steps from (1) to (4) for all the remaining discrete ordinate directions.

(6) By using Eq. (9), compute the radiative flux at each surface.

3. Results and Discussion

The method described in the analysis is applied to two examples. For the first example, the

Table 1 Effects of n and M on view factor F_{S-N}
[AJ : (Sanchez and Smith ; 1991), CH : Chevyshef weight, TP : Trapezoidal method]

n	Method	M = 2	M = 4	M = 8	M = 64	M = 512
2	AJ	0.31371	0.36020	0.36312	0.36259	0.36258
	CH	0.31371	0.36020	0.36312	0.36259	0.36258
	TP	0.41421	0.36197	0.36108	0.36255	0.36258
8	AJ	0.39769	0.42256	0.41205	0.41140	0.41141
	CH	0.39769	0.42256	0.41205	0.41140	0.41141
	TP	0.41421	0.40562	0.41417	0.41142	0.41141
16	AJ	0.40792	0.41250	0.41779	0.41345	0.41352
	CH	0.40792	0.41250	0.41779	0.41345	0.41352
	TP	0.41421	0.41094	0.41173	0.41362	0.41352
32	AJ	0.41376	0.41305	0.41317	0.41400	0.41404
	CH	0.41376	0.41305	0.41317	0.41396	0.41404
	TP	0.41421	0.41398	0.41351	0.41417	0.41404
64	AJ	0.41495	0.41354	0.41394	0.41417	0.41417
	CH	0.41495	0.41354	0.41394	0.41417	0.41417
	TP	0.41421	0.41460	0.41406	0.41419	0.41417

method is applied to compute the view factors for an infinite square duct without the protrusion. The purpose of this example is to show the accuracy of the method because the exact values are available. The emissive power of south surface (S) is assigned as 1, and the emissive powers of the other surfaces are zero. The radiative flux at each surface is the value of the view factors from the south surface (S) to the target surface (Sanchez and Smith ; 1991). The boundary surfaces

Table 2 Overall heat fluxes

(1) North wall heat flux, W/m^2

M	Sanchez. & Smith (1991)	Fiveland (1988)	Truelove (1987)	Fiveland (1984)	Chevy -shef	Trape-zoidal
2	26.654*				27.296	25.272
3	23.576*	26.235*	23.518*	23.518*	23.848	29.132
4	26.717*				26.835	26.284
6	26.676*	25.497*	23.366*		26.752	26.490
10	26.667	22.487*			26.722	26.576
15	26.687				26.528	26.723
20	26.708				26.607	26.702
	(26.603)					
50	26.727				26.657	26.655
	(26.656)*					
512	26.653*				26.654	26.654
RIM(Sanchez & Smith ; 1991)						26.657

(2) East wall heat flux, W/m^2

M	Sanchez. & Smith (1991)	Fiveland (1988)	Truelove (1987)	Fiveland (1984)	Chevy -shef	Trape-zoidal
2	18.850*				19.372	17.870
3	20.390*	19.060*	21.461*	21.461*	20.624	16.873
4	18.819*				18.968	18.621
6	18.840*	19.429*	21.333*		18.894	18.749
10	18.844	20.934*			18.850	18.823
15	18.834				18.928	18.787
20	18.824				18.853	18.837
25	18.820				18.879	18.816
	(18.876)					
50	18.814					
	(18.849)*					
512	18.851*				18.851	18.851
RIM(Sanchez & Smith ; 1991)						18.849

Numbers in parentheses are for $\alpha=0.5$. (Sanchez and Smith ; 1991). Numbers with * are results of authors program.

are divided by n. The effects of number of the discrete-ordinates method (DOM) weighting factors on F_{S-N} are presented in Table 1.

The values of the view factors F_{S-E} and F_{S-W} should be same due to geometrical symmetry. Equality of F_{S-E} and F_{S-W} are checked. Also, due to the enclosure relation (Hottel and Sarofim ; 1967), $F_{S-E}=(1-F_{S-N})/2$. Thus, only the the view factor from the south wall to the north wall, F_{S-N}, is presented in Table 1. The five significant digit exact value of F_{S-N} is equal to 0.41421.

In Table 1, the value of the view factor by the discrete-ordinate method converges to the exact value as n and M increase. For n=8 or 16, if M ≥ 8 then the relative error is within $\pm 1\%$, else within ± 5 %. If $n \geq 32$, for any M, the relative error is within ± 1 %. Thus, the accuracy of the solution depends more on the number of spatial divisions than the number of discrete-ordinate directions. Even at M=2, the vaules of the view factors by the trapezoidal method are accurate. However, when the energy balance and surface radiative flux values are checked for this case, some errors are found.

The radiative fluxes at the north and the east wall are predicted by several methods. The results are compared in Table 2. Height and width are equally divided by n=60 (Sanchez and Smith ; 1991). The temperatures of the boundary walls are $T_S=310$ K, and $T_N=T_E=T_W=300$ K. All of the boundary walls are black opaque surfaces. In this case, the radiative flux at the east wall is equal to that at the west wall.

The wall radiative flux values are computed by using the weighting factors as in Refs. (Sanchez and Smith ; 1991, Fiveland ; 1984, 1988, Truelove ;1987) and the results are tabulated in Table 2.The weighting factors in Refs. (Fiveland ; 1984, 1988, Truelove ; 1984) are used for computing the radiative heat transfer by the participating media enclosed in a three-dimensional rectangular box geometry. The results in Table 2 are the results of the code written by the authors. Only the values of the weighting factors in Refs. (Fiveland ; 1984, 1988, Truelove ; 1984) are used for the results. The value of the finite difference weighting factor α is 0.5.The number of directions per quadrant for S4, S6, S8 discrete-ordinate method is M=3, 6, 10, respectively. That $\alpha=0.6$ is utilized for the results of Sanchez and Smith (1991) in Table 2 except the results enclosed by parenthesis where $\alpha=0.5$ is used. It can be observed from the results presented in Table 2 that if $\Delta x=\Delta y$ and the point P in Fig. 2 is located at the center of the control volume, the value of finite difference weighting factor should be 0.5 to obtain the exact values by

Table 3 Effects of n and M on view factor F_{S-N}
[AJ : (Sanchez and Smith ; 1991), TP : Trapezoidal method]

n	Method	M				
		2	4	8	64	512
2	AJ	0.24264	0.29323	0.30220	0.30448	0.30451
	TP	0.41421	0.32503	0.30898	0.30458	0.30452
8	AJ	0.24032	0.33236	0.31767	0.32124	0.32124
	TP	0.41421	0.32383	0.32813	0.32123	0.32124
16	AJ	0.24982	0.31899	0.32712	0.32408	0.32417
	TP	0.41421	0.32877	0.32383	0.32427	0.32417
32	AJ	0.26147	0.32065	0.32211	0.32483	0.32490
	TP	0.41421	0.33482	0.32767	0.32502	0.32490
64	AJ	0.27046	0.32159	0.32206	0.32505	0.32508
	TP	0.41421	0.33949	0.33045	0.32518	0.32508

increasing the number of discrete angles.

In example 2, the DOM is applied to find the view factors of the system that has a protrusion as in Fig. 1. The boundary surfaces are opaque and black, and $H_1/H=0.5$, $L_1/L=0.5$. The emissive power at the south wall is 1. The emissive powers at the other five surfaces are 0. There is shading from the south wall to the east wall and to the north wall. The width and height of the enclosure are equally divided by n. In Table 3, the effect of the number of discrete directions on the view factor from the south to the north wall, F_{S-N}, is studied. The exact five digit value of F_{S-N} is 0.32514. The view factor results by the Chevyshef quadrature are the same as those presented in Table 3 of Sanchez and Smith (1991), and the results are thus omitted in Table 3. In Table 3, the value of the view factor by DOM converges to the exact values as n and M increase. If $n \geq 16$ and $M=4$, the relative error observed is within ± 5 %. If $n \geq 16$ and $M \geq 8$, the relative error observed is within ± 1 %. Therefore, if there is shading, the view factor results by DOM are sensitive both the number of discrete ordinate directions and the number of spatial divisions.

4. Conculsions

The purpose of this study is to find the view factors of an infinite rectangular duct system enclosing a transparent medium by using the discrete ordinate method. From the examples presented in this paper, conclusions are as follows. Firstly, view factors of the system with or without shadings can be obtained by using the discrete ordinate method. Secondly, as long as the shape of the control volume is square and point P in the control volume is located at the center, the finite difference weighting factor should be 0.5. If not, then the solution does not converge to the exact values. Thirdly, if there is shading in the system, both the number of spatial divisions and angular divisions have an effect on the accuracy of the solution. Not much difference is found by using the several direction weighting factors from Sanchez and Smith (1991), the Chevyshef quadrature, and the trapezoidal integration rule.

Acknowledgement

The first author would like to acknowledge the support for this study by Korea Science Foundation (#931-1000-008-2).

References

Atkinson, K. E., 1978, *An Introduction to Numerical Analysis*, John Wiley & Sons, New York.

Baumeister, J. F., 1990, "Thermal Radiation Characteristics of Nonisothermal Cylindrical Enclosures Using a Numerical Ray Tracing Technique," ASME-HTD, Vol. 137, pp. 73~79.

Chandrasekhar, S., 1950, *Radiative Transfer*, Oxford at the Clarendon Press.

Fiveland, W. A., 1984, "Discrete Ordinates Solutions of the Radiative Transport Equation for Rectangular Enclosures," *J. of Heat Transfer*, Vol. 106, pp. 699~706.

Fiveland, W. A., 1988, "Three-Dimensional Radiative Heat Transfer Solutions by the Discrete-Ordinate Method," *J. of Thermophysics and Heat Transfer*, Vol. 2, No. 4, pp. 309~316.

Hottel, H. C. and Sarofim, A. F., 1967, *Radiative Transfer*, McGraw-Hill, New York.

Kim, T. K. and Lee, H., 1988, "Effect of Anisotropic Scattering on Radiative Heat Transfer in Two-Dimensional Rectangular Enclosure," *Int. J. Heat and Mass Transfer*, Vol. 31, No. 8, pp. 1711~1721.

Kim, T. Y. and Baek, S. W., 1991, "Analysis of Combined Conductive and Radiative Heat Transfer in a Two-Dimensional Rectangular Enclosure Using the Discrete Ordinate Method," *Int. J. Heat and Mass Transfer*, Vol. 34, No. 9, pp. 2265~2273.

Naraghi, M. H. N. and Chung, B. T. F., 1984, "A Stochastic Approach for Radiative Exchange in Enclosures with Nonparticipating Medium," *Jouranl of Heat Transfer*, Vol. 106, pp. 690~698.

Sanchez, A. and Smith, T. F., 1991, "Surface Radiation Exchange for Two-Dimensional Rectangular Enclosures Using the Discrete Ordinate

Method," *J. of Heat Transfer*, Vol. 114, pp. 465 ~472.

Siegel, R. and Howell, J. R., 1981, *Thermal Radiation Heat Transfer*, Hemisphere Publishing, Washington, D. C. .

Truelove, J. S., 1987, "Discrete Ordinate Solutions of the Radiation Transport Equation," *J. of Heat Transfer*, Vol. 109, pp. 1048~1051.